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TWO PHASE FLOW ABOUT A STAGNATION  
POINT IN FILM BOILING

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JULY 1961

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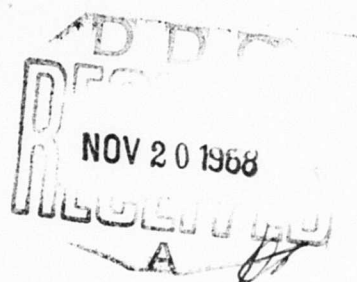
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RESEARCH NOTE 51

TWO PHASE FLOW ABOUT A STAGNATION POINT IN FILM BOILING

W. H. Gallaher

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Abstract

The analysis of film boiling of a saturated liquid about the stagnation point is presented. From this first approach macroscopic flow, an expression for the vapor layer thickness, maximum vapor velocity, and heat transfer was obtained. The effects of hydrostatic pressure gradients and radius of curvature are included. The expression for heat transfer is compared to measurement in saturated liquid nitrogen.

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## SYMBOLS

$c_p$	specific heat at constant pressure
$k$	thermal conductivity
$p$	pressure
$t$	temperature
$\dot{q}$	heat transfer rate defined by equation (8)
$u$	velocity
$V_i$	blowing velocity at the interface defined by equation (7)
$x$	distance parallel to the heated surface from the stagnation point
$y$	direction perpendicular to the heated surface
$\beta_o$	vaporization parameter
$\bar{\delta}_v$	vapor layer thickness
$\lambda$	latent heat of vaporization
$\mu$	viscosity
$\tau$	shear stress
$\rho$	density
$\alpha$	thermal diffusivity
$\gamma$	specific weight

## SUBSCRIPT

$i$	conditions at the interface
$l$	liquid layer
$o$	stagnation point conditions
$v$	vapor layer
$w$	conditions at the wall

## SUPERSCRIPT

$*$	evaluated at $t^* = \frac{t_w + t_i}{2}$
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## INTRODUCTION

Free convection heat transfer between a heated surface and a liquid is a common engineering problem. Continual heating of the surface will result in formation of bubbles, next nucleate boiling, and with sufficiently high surface temperatures, a stable vapor film can be established (see references 1 and 2, for example). This condition of a stable vapor layer between the hot solid and liquid is called film boiling.

Investigations (see references 3 and 4) have indicated that the area of the stagnation point poses an interesting problem as represented by Figure 1. Along with the two phase free convection film boiling, is an instability which is believed to initiate the interface waves found in free and forced convection film boiling<sup>3, 4</sup>.

In order to understand the physical condition of flow in the stagnation region, the analysis will be limited to a first approach macroscopic flow. By limiting the analysis further to saturated liquid and quasi-steady flow, it is possible to obtain an expression for the vapor layer thickness and heat transfer as a function of physical parameters.

### Analysis - Saturated Liquid

The following assumptions were made for the saturated liquid analysis.

1. Steady, laminar flow in both the liquid and vapor boundary layers.
2. The heated surface temperature ( $t_w$ ) is constant.

3. The temperature of the liquid-vapor interface ( $t_i$ ) is equal to the saturated liquid temperature at ambient pressure.
4. Thermal radiation effect on the vapor and liquid layer is neglected.
5. Linear temperature profile between the wall and interface.
6. Two dimensional flow.

With the above assumptions, it is possible to set up the control region as found in Figure 2. The unknowns being the vapor layer thickness, liquid boundary layer, velocity profile, pressure gradient, and heat transfer rate. The model postulated in Figure 2 is seen to have a thermally stable vapor layer with the increasing temperature gradient in the minus  $y$  direction. With such a stable system, any movement of the liquid in the  $y$  direction is to replace the liquid vaporized at the liquid-vapor interface. Assuming the liquid is at saturated temperature, indicates all energy transferred from the hot solid out of the control region to the liquid and is returned as vaporized liquid. This vapor will then be forced or induced out of the control region at some velocity  $u$ . The macroscopic control region used for this analysis has the possibility of matching the liquid velocity to the vapor layer velocity by the dynamic equilibrium equation at the interface.

$$\mu_v \left. \frac{du}{dy} \right|_{\bar{\delta}_v -} = \mu_l \left. \frac{du}{dy} \right|_{\bar{\delta}_v +} \quad (1)$$

Examination of the control region shows the vapor layer velocity  $u$  equal to zero at the wall and interface. It will be assumed the velocity  $u$  can be expressed as

$$u = a + by + cy^2 + dy^3 \quad (2)$$

which will need four conditions in order to evaluate the constants. Three boundary conditions arise with reference to Figure 2.

$$y = 0 \quad u = 0 \quad (3)$$

$$y = \bar{\delta}_v \quad u = 0 ; \quad \frac{d^2u}{dy^2} = 0 \quad (4)$$

Following a method found in reference 5, the fourth condition will be assigned by inspection;  $b = u_1/\bar{\delta}_v$  where  $u_1$  has the units of velocity. Substituting the boundary conditions and  $b = u_1/\bar{\delta}_v$  the velocity  $u$  becomes

$$u = u_1 \left[ \frac{y}{\bar{\delta}_v} - \frac{3}{2} \left( \frac{y}{\bar{\delta}_v} \right)^2 + \frac{1}{2} \left( \frac{y}{\bar{\delta}_v} \right)^3 \right] \quad (5)$$

The resulting velocity profile is found in Figure 3. The maximum value for the velocity is  $u_{\max} = 1.93 u_1$ , and occurs at  $y/\bar{\delta}_v = 0.422$ .

#### The Continuity Equation

The equation of continuity for the macroscopic control region will be the sum of the masses entering and leaving.

$$- 2 \rho_i \int_0^x V_i dx + 2 \rho^* \int_0^{\bar{\delta}_v} u dy = 0 \quad (6)$$

From an energy balance at the vapor-liquid interface, an expression for  $V_i$  is obtained.

$$V_i = \frac{k^*(t_w - t_i)}{\rho_i \bar{\delta}_v \lambda} \quad (7)$$

Equation 7 coming directly from the energy equation

$$q = \frac{k^*(t_w - t_i)}{\bar{\delta}_v} \quad (8)$$

where the heat transfer coefficient is defined as

$$h = \frac{q}{t_w - t_i} = \frac{k^*}{\bar{\delta}_v} \quad (9)$$

Limiting the analysis to a macroscopic control region will enable one to assume that  $x$  varies from 0 to  $\bar{\delta}_v$ , thereby reducing equation (6) to

$$- \rho_i V_i \bar{\delta}_v + \rho^* \int_0^{\bar{\delta}_v} u \, dy = 0 \quad (10)$$

The minus sign comes from the fact that  $V_i$  is in the minus  $y$  direction. Substitution of equation (7), (5) into (10) and integrating will give an expression for  $u_1$ .

$$u_1 = 8 \frac{k^*(t_w - t_i)}{\rho^* \bar{\delta}_v \lambda} \quad (11)$$

or defining

$$\beta_o = c_p \frac{(t_w - t_i)}{\lambda} \quad (12)$$

reported in reference 6, equation (11) can be rewritten

$$u_1 = 8 \frac{\alpha^* \beta_o}{\bar{\delta}_v} \quad (13)$$

The expression for  $u_1$ , substituted into equation (5) yields

$$u = \frac{8 \alpha^* \beta_o}{\bar{\delta}_v} \left[ \frac{y}{\bar{\delta}_v} - \frac{3}{2} \left( \frac{y}{\bar{\delta}_v} \right)^2 + \frac{1}{2} \left( \frac{y}{\bar{\delta}_v} \right)^3 \right] \quad (14)$$

where

$$u_{\max} = \frac{15.4 \alpha^* \beta_o}{\bar{\delta}_v} \quad (15)$$

Indicating  $u_{\max}$  depends on the inverse of the vapor layer thickness.

#### Vapor Layer Thickness and Pressure Gradient

Solving the momentum equation in the x direction will give an expression for the vapor layer thickness  $\bar{\delta}_v$ . At  $x = 0$ , the momentum in the x direction is zero, while at  $x = \bar{\delta}_v$  it is given as

$$\rho^* \int_0^{\bar{\delta}_v} u^2 dy \quad (16)$$



This change in momentum is equal to the principle forces acting on the control region, which will be the shear and pressure forces.

Shear force at wall:

$$\tau_w \bar{\delta}_v = - \mu_w \left. \frac{du}{dy} \right|_{y=0} \bar{\delta}_v \quad (17)$$

Shear force at interface:

$$\tau_i \bar{\delta}_v = \mu_i \left. \frac{du}{dy} \right|_{y=\bar{\delta}_v} \bar{\delta}_v \quad (18)$$

Pressure force:

$$\frac{dp}{dx} \bar{\delta}_v^2 \quad (19)$$

where  $dp/dx$  is assumed to be some function of the radius of curvature and the small component of the buoyancy force in the x-direction is neglected. Combining equations (16), (17), (18), and (19) give the momentum equation for the chosen control region.

$$\rho^* \int_0^{\bar{\delta}_v} u^2 dy + \frac{dp}{dx} \bar{\delta}_v^2 = - \bar{\delta}_v \left[ \mu_w \left. \frac{du}{dy} \right|_{y=0} - \mu_i \left. \frac{du}{dy} \right|_{y=\bar{\delta}_v} \right] \quad (20)$$

Differentiating the velocity profile equation (5)

$$\tau_w = \mu_w \left. \frac{du}{dy} \right|_{y=0} = \mu_w \frac{u_1}{\bar{\delta}_v} \quad (21)$$

$$\tau_i = \mu_i \left. \frac{du}{dy} \right|_{y=\bar{\delta}_v} = - \frac{\mu_i u_1}{2 \bar{\delta}_v} \quad (22)$$

The momentum integral is given by substitution of equation (5) into equation (16) and integrating.

$$\rho^* \int_0^{\bar{\delta}_v} u^2 dy = \frac{2}{105} \rho^* \bar{\delta}_v u_1^2 \quad (23)$$

Substitution of equations (21), (22), and (23) into (20) gives

$$\frac{2}{105} \rho^* \bar{\delta}_v u_1^2 + \bar{\delta}_v^2 \frac{dp}{dx} = -u_1 \left( \mu_w + \frac{\mu_i}{2} \right) \quad (24)$$

where

$$u_i = \frac{8 \alpha^* \beta_o}{\bar{\delta}_v} \quad (25a)$$

$$u_1^2 = \frac{64 \alpha^{*2} \beta_o^2}{\bar{\delta}_v^2} \quad (25b)$$

After substitution of (25) into (24), the momentum equation is given as

$$\frac{128}{105} \beta_o \rho^* \alpha^{*2} + \bar{\delta}_v^2 \frac{dp}{dx} = -8 \beta_o \alpha^* \left( \mu_w + \frac{\mu_i}{2} \right) \quad (26)$$

or solving for  $\bar{\delta}_v$ .

$$\bar{\delta}_v = \sqrt[3]{-\frac{1}{dp/dx} \left[ 8 \beta_o \left( \mu_w + \frac{\mu_i}{2} \right) \alpha^* + 1.22 \beta_o \rho^* \alpha^{*2} \right]} \quad (27)$$

We have an expression for the vapor layer thickness as a function of property values, physical constants, and a pressure gradient term.

It is possible to obtain the pressure gradient term from the expression for hydrostatic pressure along the model surface. Assuming the pressure gradient in the liquid boundary layer is the same as that in the vapor layer will yield the expression for conditions of Figure 4.

$$\frac{p}{\gamma} + 2z \sin^2\left(\frac{\theta}{2}\right) = \frac{p_o}{\gamma} \quad (28)$$

where

$p$  - pressure along the model surface

$p_o$  - stagnation pressure

$z$  - difference in elevation

$\theta$  - angle from the stagnation streamline

Setting the angle small ( $\sin \theta/2 = x/2R$ ) and taking the differential with respect to  $x$  we obtain

$$\frac{dp}{dx} = - \gamma \frac{x}{R^2} \quad (29)$$

For the macroscopic control region  $x = \bar{\delta}_v$  and setting  $z = R$ , equation (29) becomes

$$\frac{dp}{dx} = - \gamma \frac{\bar{\delta}_v}{R} \quad (30)$$

Substitution of equation (30) into (27) will give an expression for the vapor layer thickness assuming a hydrostatic pressure gradient

$$\bar{\delta}_v = \sqrt[4]{\frac{R}{\gamma} \left[ 8 \beta_o \alpha^* \left( \mu_w + \frac{\mu_i}{2} \right) + 1.22 \beta_o \rho^* \alpha^{*2} \right]} \quad (31)$$

indicating the vapor layer thickness is a direct function of the radius  $R$  and property values.

Plotting equation (31) against  $\beta_o$  for radii is found in Figure 5. Increasing  $\beta_o$  and radius will result in larger values for the vapor layer thickness  $\bar{\delta}_v$ .

Using equations (13) and (31) it is possible to obtain the maximum vapor layer velocity at  $x = \bar{\delta}_v$  as shown in Figure 6. The maximum velocity at  $x = \bar{\delta}_v$  is found to increase with  $\beta_o$  and decrease with increasing radius for constant  $\beta_o$ . The value of the maximum velocity is found to be in the 0-5 ft/sec range.

Conduction heat transfer rate as a function of  $\beta_o$  and radius  $R$  can be obtained from equations (8) and (31). Comparing the heat transfer rate with experimental information for saturated liquid nitrogen is found in Figure 7. For a constant  $\beta_o$ , the heat transfer rate increases with decreasing radii. Substantial increases in heat transfer rate occur for radii smaller than 1 inch.

The experimental points for saturated nitrogen (see reference 3) are lower than the present theory predicts. Two possible explanations are the manner in which the data was obtained and the error involved when

comparing a two dimensional theory to a three dimensional experiment. Experimentors of reference 3 used a model having a flat end as the stagnation region. It seems likely that the heat transfer data would have an "effective radius" larger than the actual radius of the rod. Referring to Figure 7 the model appears to have an effective radius of about 1 inch, whereas the actual radius is 0.313 inch.

### DISCUSSION AND RESULTS

The present analysis is based in part on two assumptions which may change with a more rigorous analysis: the velocity profile and pressure gradient. However from the restrictive two dimensional physical model, it was possible to obtain an expression for a quasi-steady vapor layer thickness which depended upon  $\beta_0$  and a radius  $R$ .

Applying the vapor layer thickness to the derived equation for vapor layer maximum velocity found values between 0 and 5 ft/sec. This indicates convection plays a minor role in the stagnation point region heat transfer and validates the assumption of a linear temperature distribution.

Heat transfer data in the stagnation point region can be made to agree with the present theory, if an effective radius of curvature is used. Without the assumption of an effective radius, the experimental data of reference 3 is 30% below the present theory. It is felt that the present theory should be expanded into a three dimensional theory and to include the

subcooled liquid film boiling case. Expanding into the subcooled case would allow evaluations of the assumptions since the bulk of experimental data is for the subcooled liquid case.

#### ACKNOWLEDGEMENT

The writer wishes to express his appreciation to W. S. Bradfield for his assistance as adviser to this report.

## REFERENCES

1. Jakob, M., Heat Transfer Vol. 11, John Wiley & Sons, New York, 1949.
2. Krieth, F., Principles of Heat Transfer, Scranton International Text Book Co., 1958.
3. Bradfield, W. S., Barkdoll, R. O., Byrne, J. T., Film Boiling on Hydrodynamic Bodies, Convair Scientific Research Laboratory, RN-37, December 1960.
4. Petersen, J. F., Interface Waves at a Stagnation Point in Free Convection Film Boiling: An Experimental Study, Convair Scientific Research Laboratory, RN-49, June 1961.
5. Eckert, E. R. G., Drake, R. M., Heat and Mass Transfer, McGraw-Hill Book Co., New York, 1959.
6. Bradfield, W. S., Plane Laminar Forced Convection Film Boiling with Subcooling, Convair Scientific Research Laboratory, RN-35, July 1960.

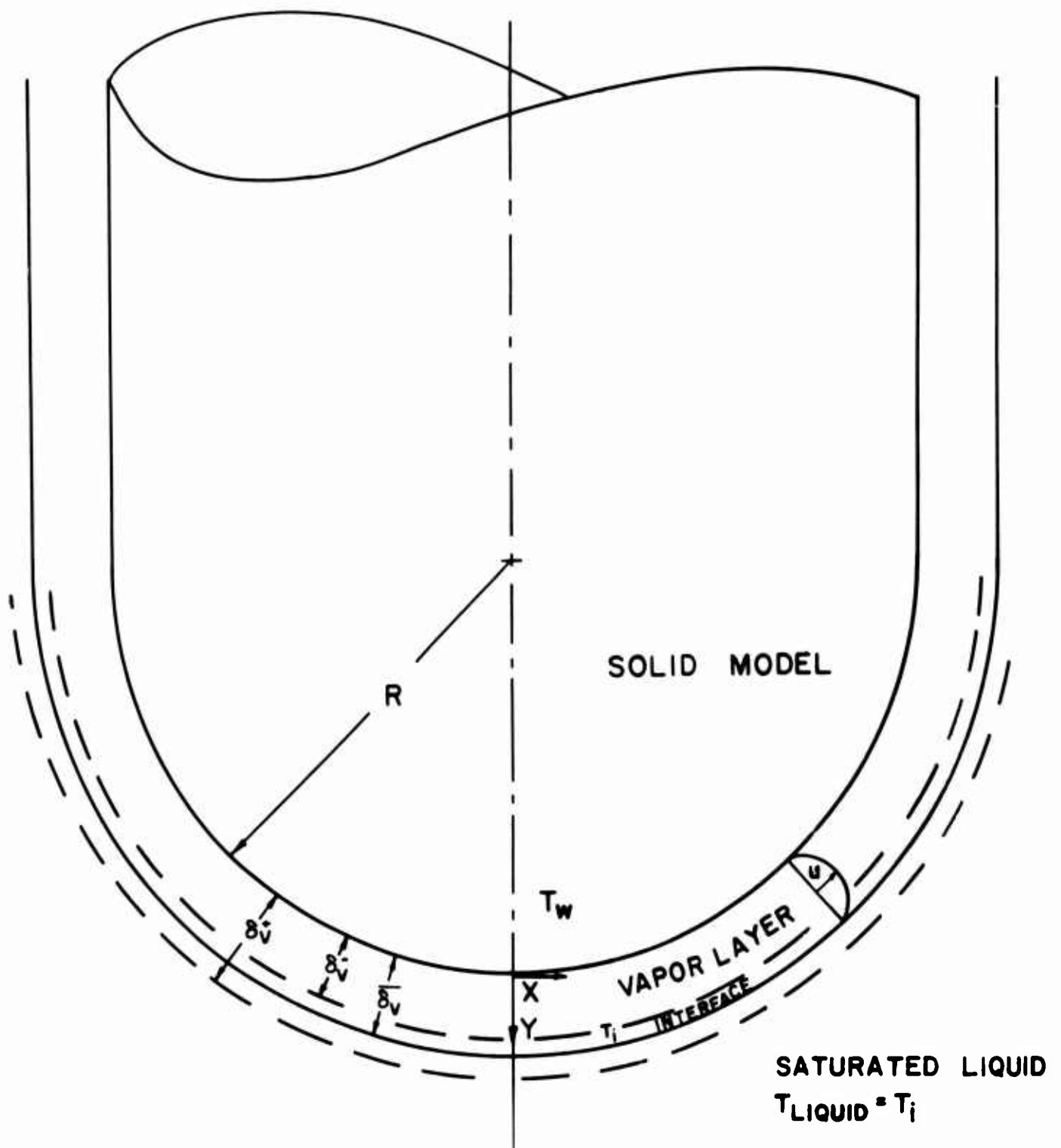
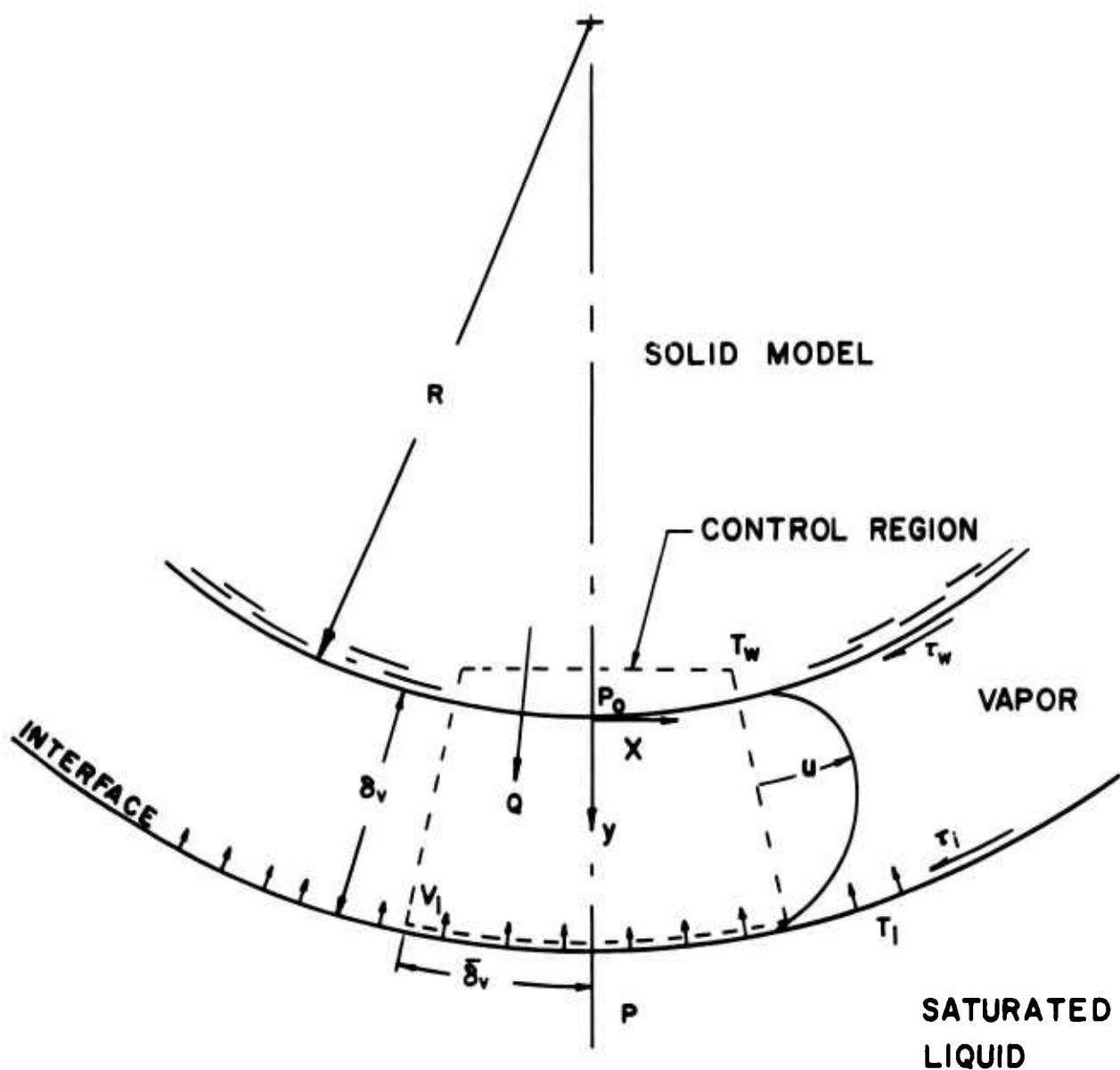


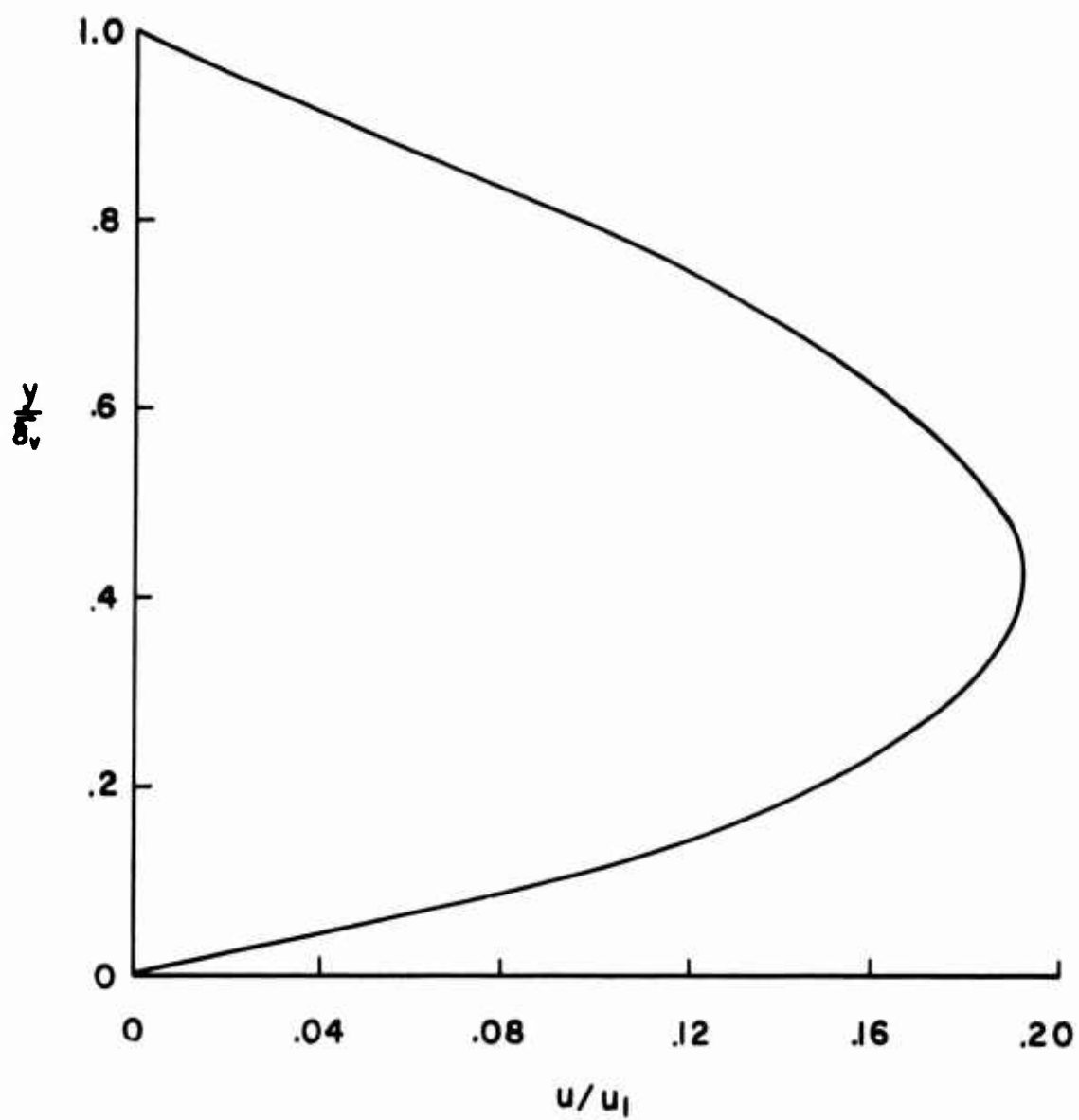
FIG. 1 FREE CONVECTION FILM BOILING IN THE STAGNATION POINT REGION





**FIG. 2**      **MACROSCOPIC CONTROL REGION FOR TWO PHASE FLOW ABOUT A STAGNATION POINT IN FILM BOILING**

FIG. 3 VELOCITY PROFILE OF THE VAPOR LAYER



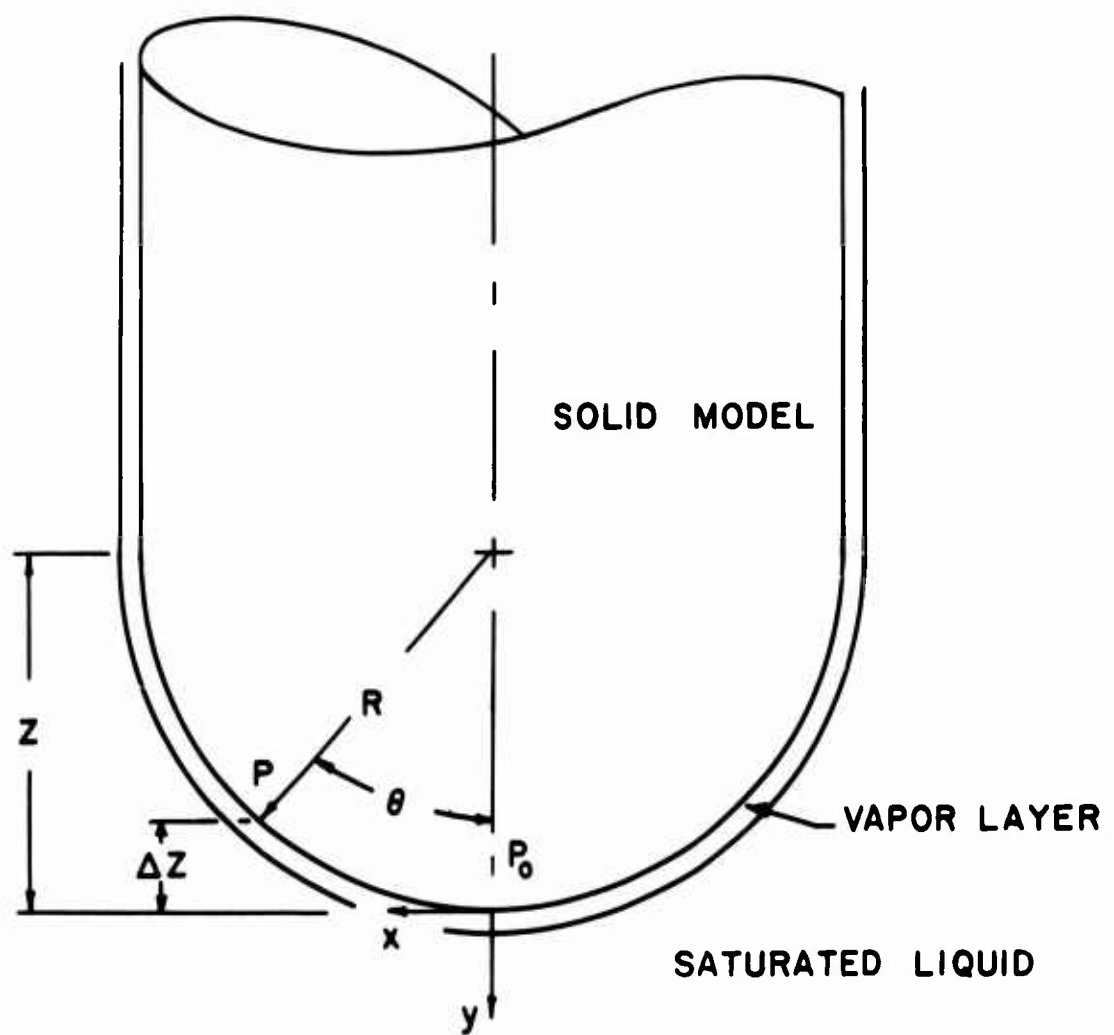


FIG. 4 HYDROSTATIC PRESSURE ALONG THE MODEL SURFACE

SATURATED NITROGEN FILM BOILING  
VAPOR LAYER THICKNESS -  $\bar{\delta}_v$

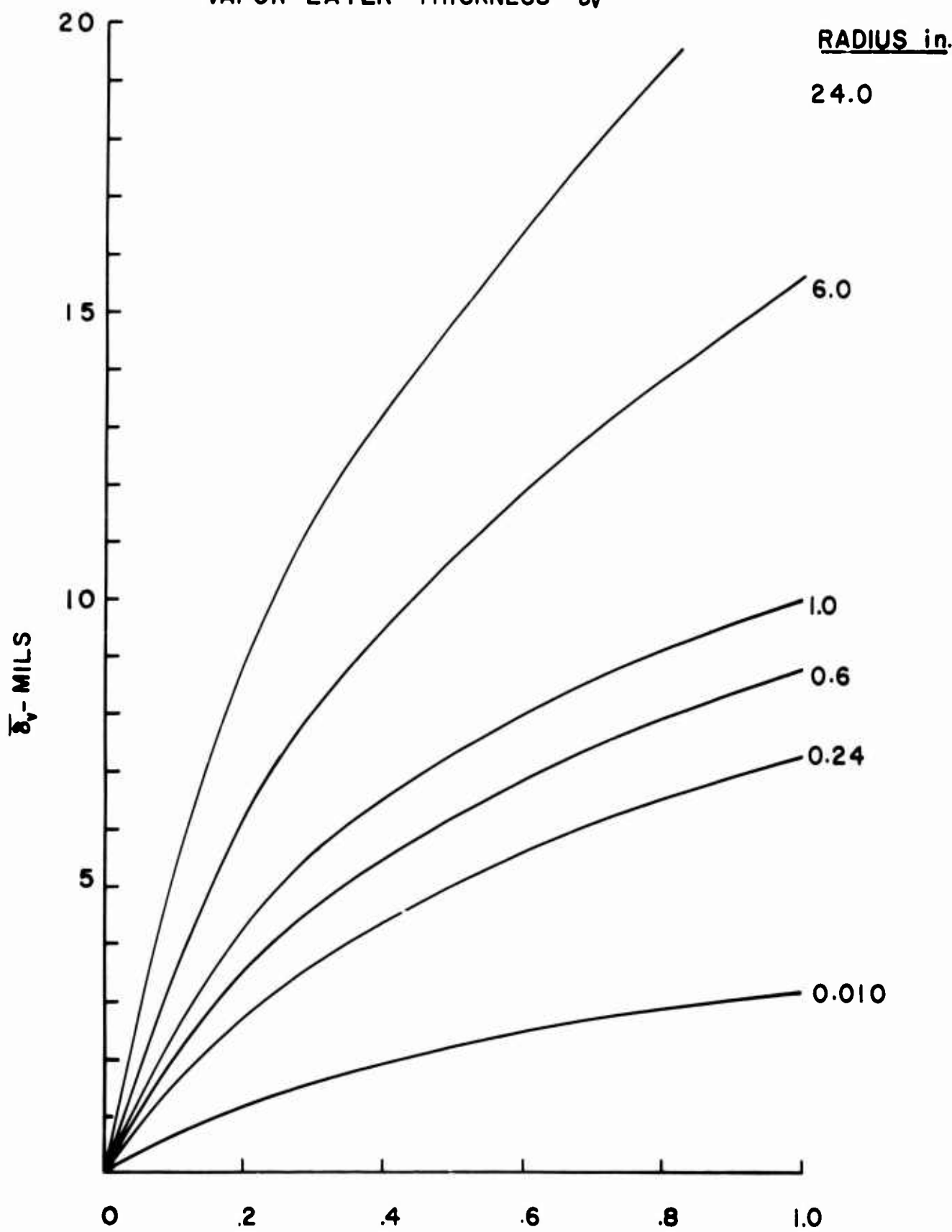


FIG. 5 VAPOR LAYER THICKNESS IN SATURATED NITROGEN

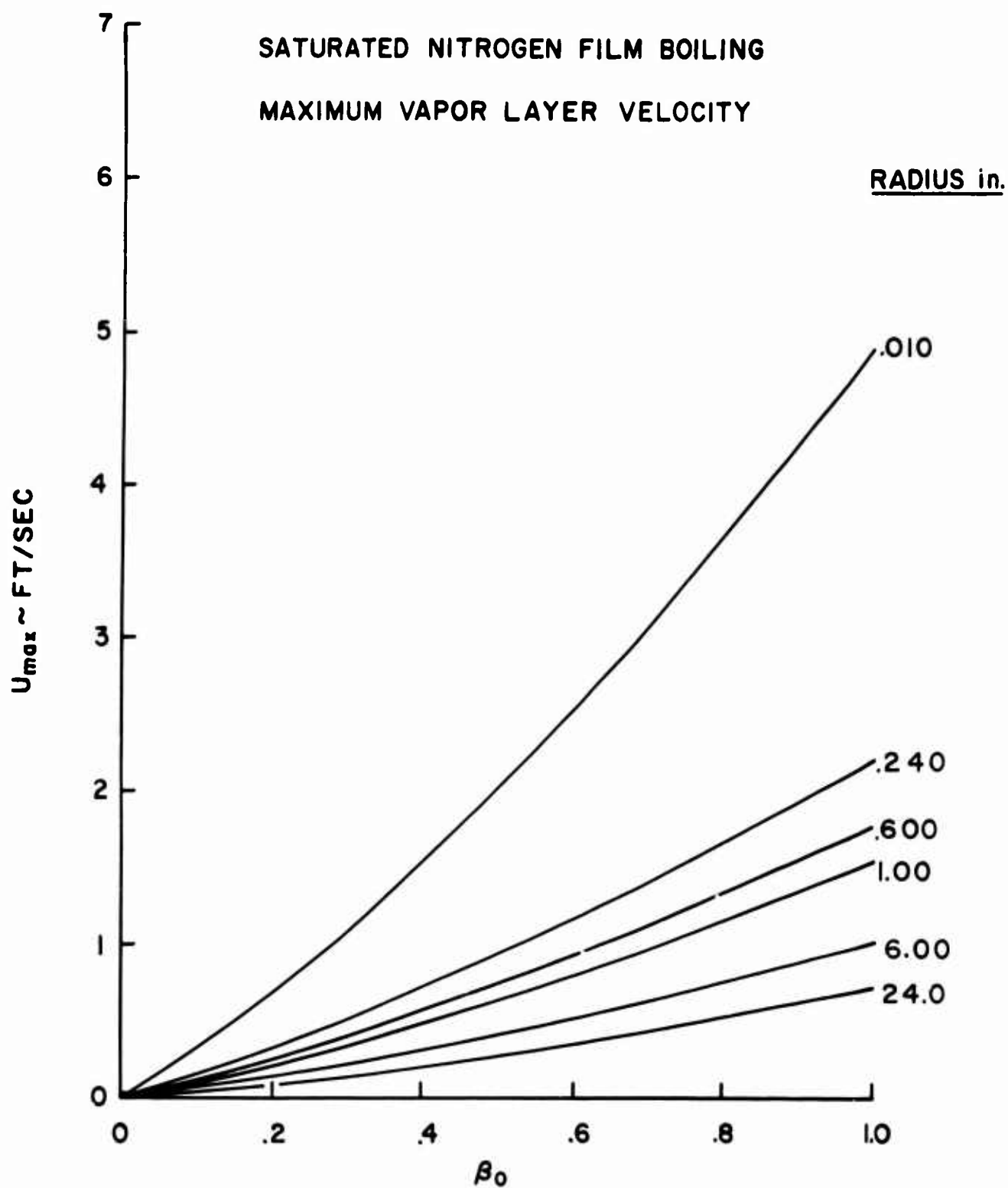


FIG. 6 MAXIMUM VAPOR LAYER VELOCITY VS VAPORIZATION  
PARAMETER

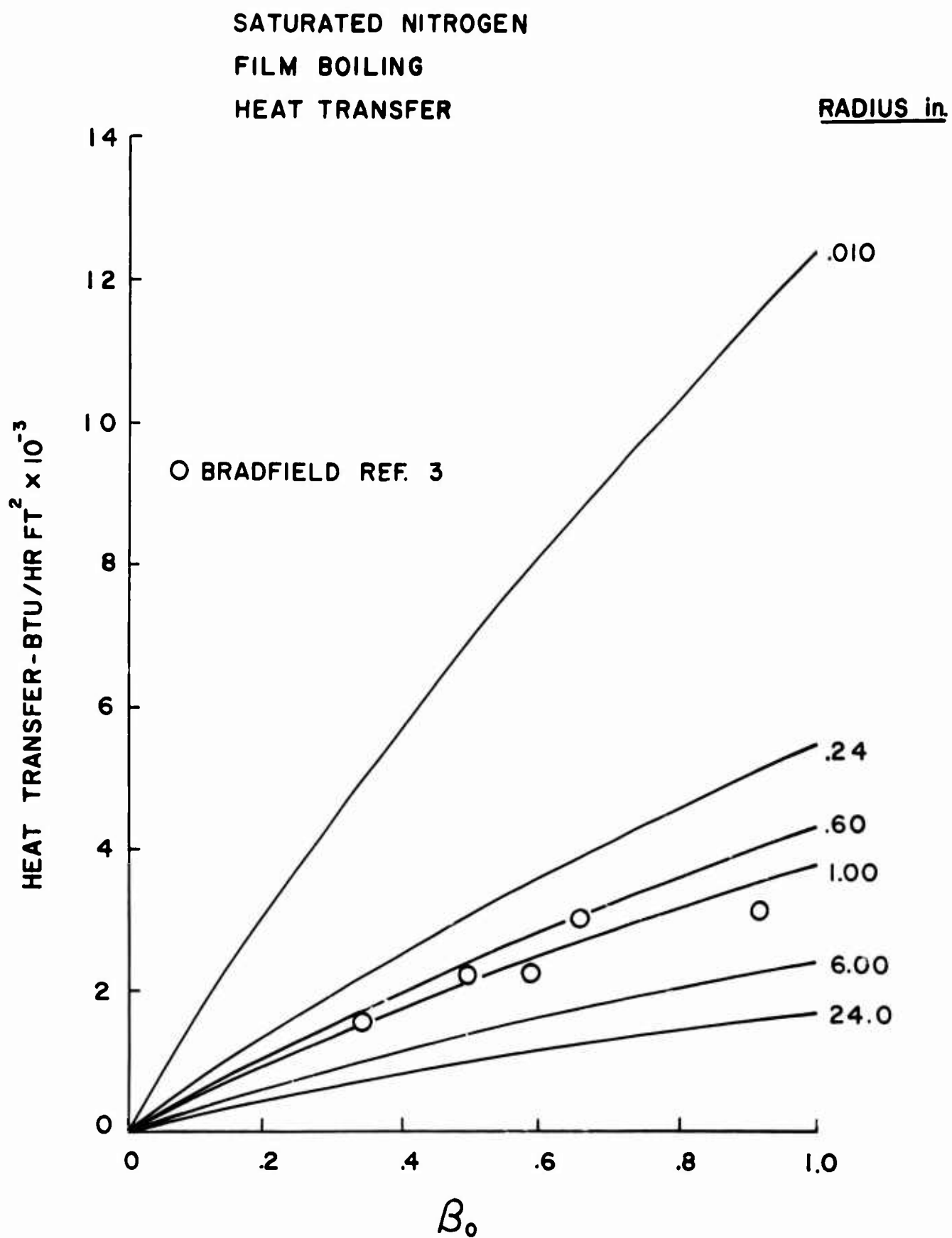


FIG. 7 SATURATED NITROGEN HEAT TRANSFER VS  
VAPORIZATION PARAMETER